

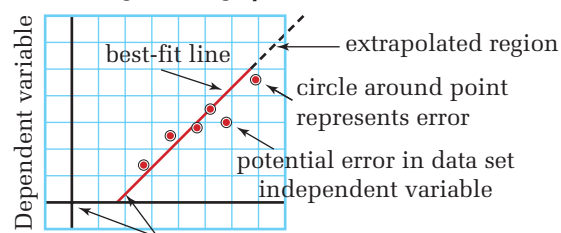
Drawing and Interpreting Graphs

Graphical analysis of scientific data is used to determine trends. Good communication requires that graphs be produced using a standard method. Careful analysis of a graph could reveal more information than the data alone.

Standards for Drawing a Graph

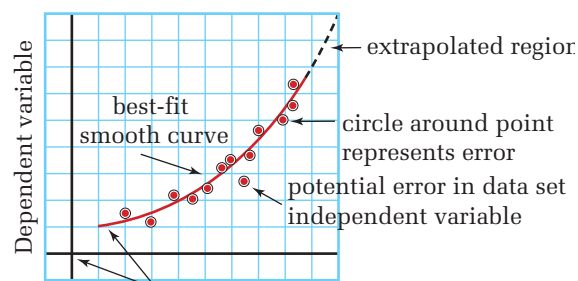
- Independent variable is plotted along the horizontal axis (include units).
- Dependent variable is plotted along the vertical axis (include units).
- Decide whether the origin (0,0) is a valid data point.
- Select convenient scaling on the graph paper that will spread the data out as much as possible.
- A small circle is drawn around each data point to represent possible error.
- Determine a trend in the data — draw a best-fit line or best-fit smooth curve. Data points should never be connected directly when finding a trend.
- Select a title that clearly identifies what the graph represents.

Constructing a linear graph



Never “force” a line through the origin.

Constructing a non-linear graph



Never “force” a line through the origin.

Interpolation and Extrapolation

A best-fit line or best-fit smooth curve that is extended beyond the size of the data set should be shown as a dashed line. You are extrapolating values when you read them from the dashed-line region of the graph. You are interpolating values when you read them from the solid-line region of the graph.

Find a Trend

The best-fit line or smooth curve provides insight into the type of relationship between the variables represented in a graph.

A *best-fit line* is drawn so that it matches the general trend of the data. You should try to have as many points above the line as are below it. Do not cause the line to change slope dramatically to include only one data point that does not seem to be in line with all of the others.

A *best-fit smooth curve* should be drawn so that it matches the general trend of the data. You should try to have as many points above the line as are below it, but ensure that the curve changes smoothly. Do not cause the curve to change direction dramatically to include only one data point that does not seem to be in line with all of the others.

Definition of a Linear Relationship

A data set that is most accurately represented with a *straight line* is said to be linear. Data related by a linear relationship can be written in the form

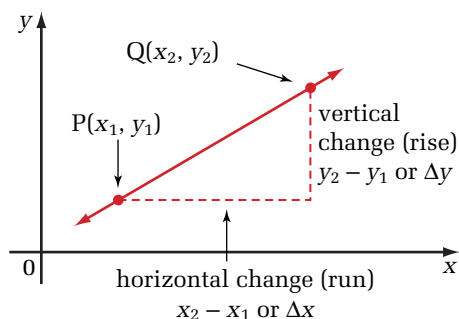
$$y = mx + b$$

Quantity	Symbol	SI unit
y value (dependent variable)	y	obtained from the vertical axis
x value (independent variable)	x	obtained from the horizontal axis
slope of the line	m	rise/run
y-intercept	b	obtained from the vertical axis when x is zero

continued ►

Slope (m)

Calculating the slope of a line



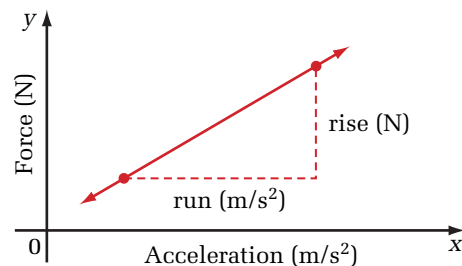
$$\text{slope } (m) = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$$

Mathematically, slope provides a measure of the steepness of a line by dividing the vertical change (rise) by the horizontal change (run). In scientific situations, it is also very important to include units of the slope. The units will provide physical significance to the slope value.

For example:



Including the units throughout the calculation helps verify the physical quantity that the slope represents.

$$m = \frac{\text{rise (N)}}{\text{run (m/s}^2\text{)}} \quad \text{Recall : } 1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

$$m = \frac{\text{kg} \cdot \cancel{\text{m/s}^2}}{\cancel{\text{m/s}^2}}$$

$$m = \text{kg}$$

In this example, the slope of the line represents the physical quantity of mass.

Definition of a Non-Linear Relationship

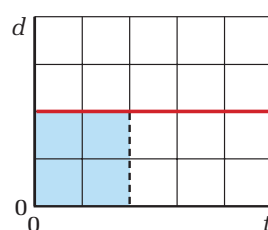
A data set that is most accurately represented with a smooth curve is said to be non-linear. Data related by a non-linear relationship can take several different forms. Two common non-linear relationships are as follows.

(a) parabolic $y = ax^2 + k$

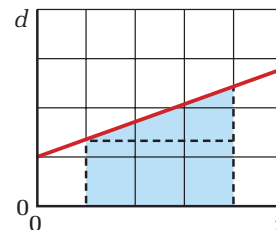
(b) inverse $y = \frac{1}{x}$

Area Under a Curve

Mathematically, the area under a curve can be obtained without the use of calculus by finding the area using geometric shapes.

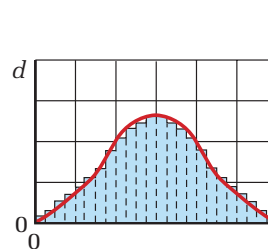


Total area = length \times width

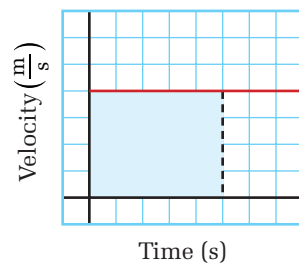


Total area =
area of the rectangle +
area of the triangle

Always include units in area calculations. The units will provide physical significance to the area value. For example, see below.



Total area =
area 1 + area 2 + area 3 ...



Including the units throughout the calculation helps verify the physical quantity that the area represents.

$$\text{Area} = (\text{length})(\text{width})$$

$$\text{Area} = (\text{velocity})(\text{time})$$

$$\text{Area} = (\text{m/s})(\text{s})$$

$$\text{Area} = \text{m (base unit for displacement)}$$

The units verify that the area under a speed-versus-time curve represents displacement (m).

1. (a) Plot the data in Table 1 by hand, ensuring that it fills at least two thirds of the page and has clearly labelled axes that include the units.
- (b) Draw a best-fit line through the plotted data.
- (c) Based on the data trend and the best-fit line, which data point seems to be most in error?
- (d) Interpolate the time it would take to travel 14 m.
- (e) Extrapolate to find how far the object would travel in 20 s.

Table 1

Time (s)	Distance (m)	Time (s)	Distance (m)
0	2	8	17
1	4	9	20
2	7	10	23
3	8	11	24
4	5	12	26
5	12	13	29
6	16	14	28
7	16	15	33

2. (a) Plot the data in Table 2 by hand, ensuring that it fills at least two thirds of the page and has clearly labelled axes that include the units.
- (b) Draw a best-fit smooth curve through the plotted data.
- (c) Does this smooth curve represent a linear or non-linear relationship?
- (d) At what force is the position at the greatest value?

Table 2

Force (N)	Position (m)	Force (N)	Position (m)
0	0.0	1.1	2.5
0.1	0.5	1.2	2.5
0.2	0.9	1.3	2.4
0.3	1.3	1.4	2.2
0.4	1.6	1.5	2.0
0.5	1.9	1.6	1.7
0.6	2.1	1.7	1.4
0.7	2.3	1.8	1.1
0.8	2.4	1.9	0.7
0.9	2.5	2	0.2
1	2.6		

3. Find the area of the shaded regions under the following graphs. Use the units to determine the physical quantity that the area represents.

